# Kerr black hole singularity 3D embedding

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## 1 Analytical embedding

We assume $[x(\theta,\phi), y(\theta,\phi), z(\theta)] := [\rho(\theta)\cos\phi, \rho(\theta)\sin\phi, z(\theta)]$ that respects the Euclidean metric

$$ds^2 = dx^2 + dy^2 + dz^2 = [\ \rho'(\theta)^2 + z'(\theta)^2\ ]\ d\theta^2 + \rho(\theta)^2 d\phi^2 \tag{1.0.1}$$

and we take in exam the hypersurfaces $t = const. \wedge r = const. = 0$, which means considering the region of a Kerr black hole containing the *physical singularity* in a fixed instant. This leads to

$$dt = dr = 0. \tag{1.0.2}$$

In doing so, the line-element (in Boyer-Lindquist coordinates) is reduced to:

$$ds^2 = a^2\cos^2\theta\ d\theta^2 + a^2\sin^2\theta\ d\phi^2 \tag{1.0.3}$$

where

$$a^2\cos^2\theta = \bar{g}_{22}(r=0,\theta)$$
$$a^2\sin^2\theta = \bar{g}_{33}(r=0,\theta).$$

So matching the metric we have:

$$\rho(\theta) = a\sin\theta \mid \rho'(\theta) = a\cos\theta\ \wedge\ z'(\theta) = 0 \tag{1.0.4}$$

or rather, the vector

$$[x(\theta,\phi), y(\theta,\phi), z(\theta)] := [a\sin\theta\cos\phi, a\sin\theta\sin\phi, z_0]. \tag{1.0.5}$$

Of course such coordinates are well defined in the point $(r = 0, \theta = \frac{\pi}{2})$[(1)]. Thus setting $\theta = \frac{\pi}{2}$ we obtain the singularity shape

[(1)] note that to do this, we did not have to resort to the Kerr-Schild coordinates.

$$[x(\tfrac{\pi}{2}, \phi), y(\tfrac{\pi}{2}, \phi), z(\tfrac{\pi}{2})] := [a \cos\phi, a \sin\phi, z_0] \tag{1.0.6}$$

i.e.

$$x^2 + y^2 = a^2 \tag{1.0.7}$$

that is a ring of radius $a$. And here we find the most significant result: the *singularity* mantains its *ring structure* even in a 3D flat Euclidean space. And what's even more incredibile, is that the radius of this ring is the *angular momentum* of the rotating black hole itself, exactly as it happens with the Kerr-Schild non-physical coordinates. This must make us reflect on how and on how much the concepts of *energy* and *geometry of the space-time* are inseparably linked in general relativity.

## 2 Embedding plot for a=0.64

In order to be consistent with the previous article, we set $a = 0.64$. I warn in advance that no notable interal region has been plotted: infact, the embedding process of inner event horizon (Cauchy horizon) and ergosphere hypersurfaces deserves a separate article and discussion. All the graphs have been translated so as to be symmetrical with respect to origin.

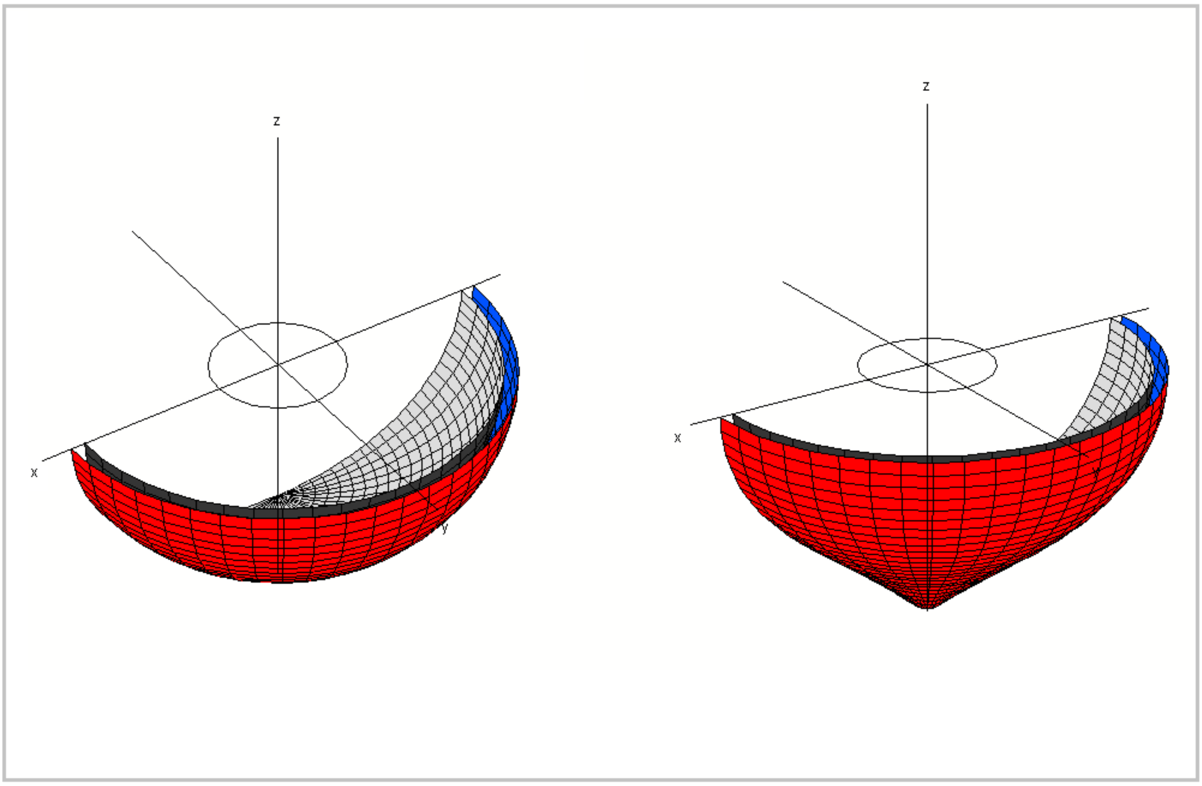


Figure 1: *The interior of a Kerr rotating black hole from two different angles. The outer ergosphere and event horizon are sectioned in order to make the singularity ring more visible. No other notable inner region has ben plotted here.*

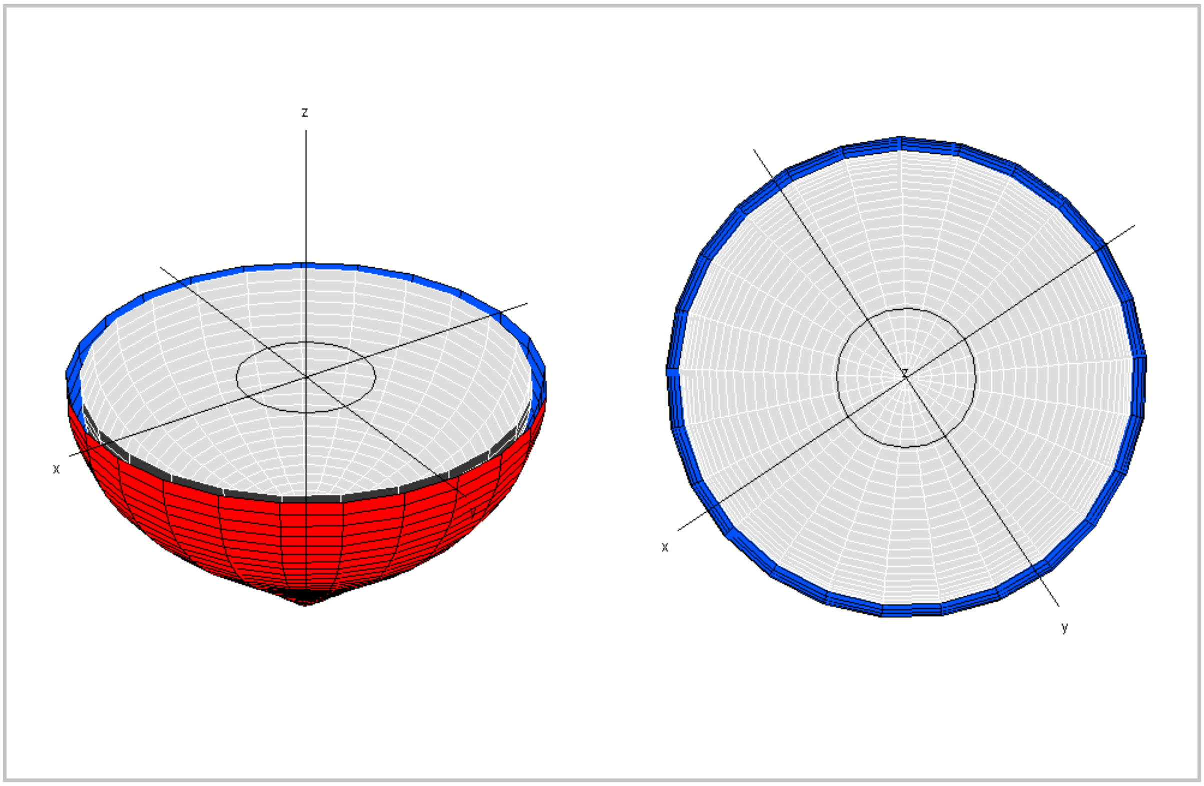


Figure 2: *Kerr black hole interior for $z \leq 0$. No other notable inner region has been plotted here beyond singularity.*

## 3 Conclusions

Of course, we should not expect the inside of a real black hole to be done this way for several reasons: first of all, we know that there are various energetic motivations for which the inner regions, in a stellar collapse situation, would collapse in turn(2). But apart from that, one must think that from our flat space it's not possible to observe what is within the outer event horizon, since no information can escape from it; instead, if we physically go beyond the horizon itself, the space would certainly not be flat. Thus, countrary to what happened for outer ergosphere and event horizon, which constitute possibile region to be observed directly, the figures we see above show what shape the singularity would take if it were properly "transported" in our Euclidean 3D space, in a completely theoretical way.

(2)there is some physical way for the structure to hold up, and this is related to quantum gravity.